Equations to Determine Energy Losses in Sudden and Gradual Change of Direction

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Abstract Equations to estimate the *K* coefficient have been obtained for devices of change direction of ducts under forced regime. For this approach, a state of art was carried out where it was noticed that the traditionally utilized methods that evaluate the losses of the devices needed tables and/or graphics when estimating the *K* coefficient, which is singular to each accessory. Each theory was analyzed and classified accordingly to each piece whenever these are present in sudden or gradual conditions. The data was homogenized with the purpose to obtain average curves values of the coefficient. The results were used as data in methods of multiple linear regressions until obtain a representative equation for each case studied. These equations have the advantage of being reliable to determine the loss of *K* coefficient without having to manipulate tables and graphics; this allows saving time when designing and testing the hydraulic behavior in forced ducts. Finally, these equations can be implemented in advanced computational algorithms which will allow the analysis and modeling of the losses caused by friction in different scenarios.

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1 Introduction

The loss of hydraulic energy is vital to quantify, because it is necessary to know the amount of available hydraulic energy in order to use it in a system such as: the conveying lines in industrial plants (Anaya-Durand et al. 2014), internal distribution networks in geothermal centrals (Di Maria 2000), ducts in hydroelectric plants (Elbatran et al. 2015), water distribution networks to populations (Yıldırım and Singh 2010), drip irrigation systems (Sesma et al. 2015), and many more which can be cited.

The study of energy losses is divided in: friction losses and local losses, both having a different origin (Liu et al. 2013), and with reference to the latter the present work is concentrated. Local losses take place when the fluid flows in a duct which has a constant diameter and suddenly, the duct changes direction or it exist an expansion, reduction, bifurcation, or obstruction (accessory). This generally happens in the surroundings of the element that causes it and its magnitude, origins and quantification depends of the specific characteristics of the device (Bariviera et al. 2013; Fuentes and Rosales 2004). Our investigation focuses on the study of energy losses caused by a change in direction of the flow (curves) of circular conduits, being this device the highly used in the building of forced conductions.

A change in direction can occur in sudden or gradual conditions. In a sudden direction change, the direction of the fluid is abruptly modified which causes a detachment or separation of the fluid in the internal part of the curve and a pressure increase in the external part caused by the intensity of the impact (Fig. 1a). This causes that, after the curve the motion of the flow change into a spiral form up to lengths equal 50 times the diameter of the conduction (Hellström et al. 2013; Sotelo 2013). However, when the direction change is performed in gradual form (Fig. 1b), the phenomenon of the fluid's alteration is similar to the abrupt change, but it happens in lower magnitude according to how smooth the curve is produced (Sotelo 2013).

Recent investigations carried out by: Csizmadia and Hős (2014), Gasljevic and Matthys (2009), Hellström et al. (2013), Ji et al. (2010), Noorani et al. (2013),

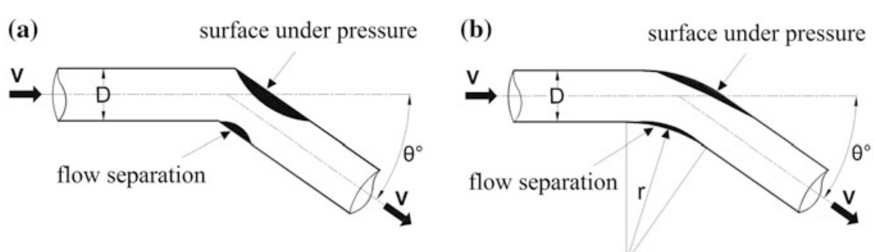

Fig. 1 Behavior of circulating flow in: **a** Sudden change of direction. **b** Gradual change of direction. θ is the deflection angle of the *curve*, r is curvature's ratio and D the diameter of the conduit

Shi et al. (2013), Ulusarslan (2010); among others, support the importance of studying the phenomenon and the way energy losses in a change of direction are quantified. Despite having an array of analytical and experimental results to obtain the losses of K coefficient for this device, there is no generalized value for the geometrical and physical conditions on which these devices are being used.

In this investigation are obtained equations to determine the K coefficient of a sudden and gradual change of direction, providing an average value using experimental information.

2 Theory

2.1 Estimation of Energy Losses

In literature (Acero and Rodríguez 2008; Bariviera et al. 2013; Sotelo 2013; Streeter et al. 2000; USACE 1980; USBR 1985; Yıldırım and Singh 2010)—among others—local energy losses have been determined as follows:

$$h_L = K \frac{V^2}{2g} \tag{1}$$

where: h_L is the loss of hydraulic charge (m); g is the acceleration of gravity (m/s^2); V is the average speed of flow (m/s) which, in a change of direction is the one produced in the upstream above the accessory; K is the coefficient of losses in the accessory (dimensionless) which is obtained by employing tables or graphics from experimental results who are often combined with empirical equations whose values are dependent to physical and geometrical parameters and, in occasions of the Reynolds Number (Bae and Kim 2014). This number is calculated as follows:

$$R_e = \frac{DV}{v} \tag{2}$$

where: R_e is the Reynolds Number (dimensionless); D is the diameter of the conduit (m); V is the average speed of flow (m/s); v is t kinematic viscosity of the fluid (m^2/s).

2.2 Estimation of the K Coefficient

(*a*) *Sudden Change of Direction*
Sotelo (2013) and USACE (1980) recommended the use of a graphical method to estimate the K coefficient, using the laboratory results of Kirchbach and Schubart. This graphic has curves for three Reynolds numbers (20000, 60000, and 250000) and relate to the deflection angle (θ) to obtain the coefficient of interest. On the other hand,

in CFE (1983) and Miller (1978), the K losses coefficient is obtained from a graphic for $R_e = 1000000$, where the value of K is subjected to the deflection angle (θ_b).

(*b*) *Gradual Change of Direction*
In USACE (1980), it is said that the coefficient of loss can be estimated using a Wasieliewski's graphic which depends on the deflection of the angle (θ) and to the relation of the curve's ratio according to the diameter of the device (r/D). Based on USBR (1985), it has been suggested a graphic to obtain the experimental coefficient value when the central angle of the curve is 90°. This graphic has several curves proposed by various researchers and a curve fitting proposed by the author of this paper which will be also considered as part of the article itself. To this point, the losses coefficient depends on the ratio of the curvature and the diameter of the accessory (R_b/D). Subsequently, the author proposes other graphic with a correction factor for a change in direction with other curvature angle different of 90°. In order to obtain the most adequate K coefficient, the correction factor proposed by the author is proportional to the resulting coefficient of the curve with an angle of 90°. In Sotelo (2013), the K coefficient can be determined as follows:

$$K = C_c \frac{\theta^\circ}{90^\circ} \tag{3}$$

where; K is the experimental coefficient; θ is the inner angle of the curvature, and C_c is a coefficient for curves presenting a constant diameter and for the Reynolds number $>2.2 \times 10^5$. This number has been obtained from the Hoffman graphic which depends the ratio of the curve and the diameter of the conduit (r/D), and at the same time, it depends on the relation between the roughness and the diameter (ε/D). In order to obtain the same amount of parameters, this research considers the average curve of ε/D. Considering CFE (1983) and Miller (1978), the experimental coefficient can be obtained in a graphic representation when the flow reaches its standarization and the Reynolds' number is equal to 10^6. On this graphic, the coefficient is governed by the deflection angle and the relation between the ratio of the curve and the diameter of the conduit (r/D). The authors of the current research suggest three other graphics where correction coefficients can be obtained for the Reynolds Number, by the length after the curve, and by the roughness of the conduit. However, this current research will not consider these corrections in order to maintain a degree of uniformity on the parameters regarding the other authors. According to SARH (1984), the K coefficient in a change of direction, presenting a deflection of 90°, can be estimated through a tabulation which is governed by the relation between the ratio of the curvature and the diameter of the conduit (r/D). Nevertheless, if the deflection is different from 90°, the obtained coefficient should be proportional to a η factor which can be determined with a second tabulation and it will depend on the magnitude of the deflection angle (θ).

3 Estimation and Analysis of the K Coefficient

Based on what has been found in the literature, the study of energy losses on devices which have direction changes are divided according to sudden conditions and gradual conditions. Based on this criterion.

(*a*) *Sudden Change of Direction*
In order to estimate the K coefficient in sudden conditions, the methods suggested in CFE (1983), Miller (1978), Sotelo (2013) and USACE (1980) were choose to work with. All of them employ graphics to estimate the losses coefficient dependent to the deflection angle (θ). Using the graphs, coordinates were obtained showing the points on the curves; afterwards, this information was homogenized in such manner that, for each method, values of the K coefficient were obtained for values of θ. from 0 to 90°, in 5° increments. Based on the obtained data, using the previously described process, average values were estimated and subsequently proposed in this research, indicated as K_{CB}. Figure 2 shows the behavior of the values—represented by curves-of the K coefficient; the coefficients and the curve of the average losses coefficient (K_{CB}) were determined by the employment of the previously selected methods. Furthermore, this figure shows the tendency of the K coefficient of a given value of θ and R_e, whereas the K_{CB} coefficient depends just to the angle θ.

A multiple linear regression was carried out using the values of the K_{CB} coefficient. Figure 3 shows the curve fitting and its equation in order to estimate the coefficient of a device which makes an abrupt direction change (K_{CB}) with a confidence of 99.9 %

$$K_{CB} = \frac{0.0031960558 + 0.0030444516\,\theta}{1 - 0.014390831\,\theta + 0.00006719314\,\theta^2} \tag{4}$$

where: K_{CB} is the coefficient of an abrupt change of direction; θ is the deflection angle of the curve (degrees). In order to apply this equation it suffices to know the angle θ. This equation is suggested to obtain the values with previously exposed level of trust/confidence only in the cases when the angle is greater than 0 and lesser or equal to 90°.

(*b*) *Gradual Change of Direction*
In order to estimate the K coefficient in gradual conditions, methods proposed in CFE (1983), Miller (1978), SARH (1984), Sotelo (2013), USACE (1980) and USBR (1985) were employed; these methods involved graphics, tables, and equations.

Values of the losses coefficient were obtained for each method in a relation to the curvature ratio regarding to a diameter (*r/D*) of 1 and to values of the deflection angle (θ) from 0 to 90° in steps of 5°. K values were obtained in a similar way corresponding to the relations of *r/D* 1.5, 2, 4, 6, and 10. Once having these results, average values of the losses coefficient were determined for each value of θ for *r/D* = 1; each increment is represented as K_{CG1}. This average was also carried out

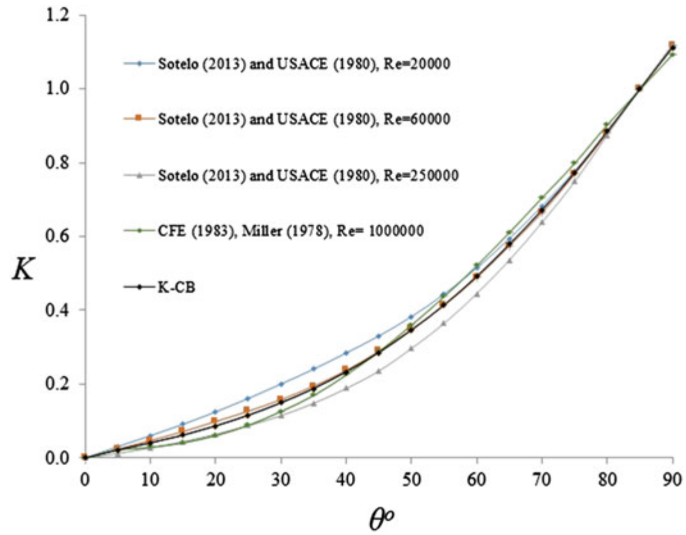

Fig. 2 Curves of the K coefficient in an abrupt change of direction corresponding to the angle of the deflection curve in degrees. The *K-CB* curve represents the average values

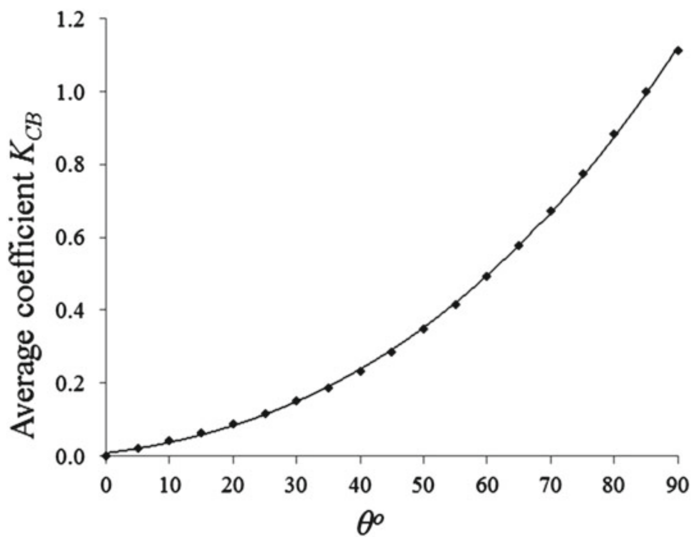

Fig. 3 Fitting curve and the equation of the K_{CB} coefficient regarding the deflection angle of the curve (θ)

in the case that r/D might be of 1.5, 2, 4, 6, and 10; these average values are represented as K_{CG2}, K_{CG3}, K_{CG4}, K_{CG5}, and K_{CG6}, respectively, Fig. 4 shows the tendency of the average values of the K_{CG} coefficient with respect to the variation of the curve's deflection angle (θ).

Once all the average values were obtained, a multiple linear regression was carried out where a fitting curve was selected for the 6 classifications of K_{CG}, i.e. a fitting curve of 99.9 % was obtained for the K_{CG1} values (Fig. 5). This resulted in the same curve for the other 5, but with a different fitting on each one of them.

In general, the equation of the fitting curve for all 6 classifications of K_{CG} can be represented as follows:

$$K_{CG} = \frac{(a + c\theta)}{(1 + b\theta)} \tag{5}$$

where: K_{CG} is the losses coefficient in gradual direction change with a fixed value of *r/D*; *a, b, and c* are constants that vary in relation to *r/D*; θ is the value of the deflected angle in degrees. Since the values of the constants (*a, b, c*) vary on each classification of the K_{CG} coefficient, a fitting curve was defined for each of the constants regarding the relation *r/D*. Figure 6 shows the fitting curve with the values of constant *a* and the Eq. (6) which defines its value with a 99.9 % confidence. Equations for constant *b* and *c* are represented in (7) and (8), respectively.

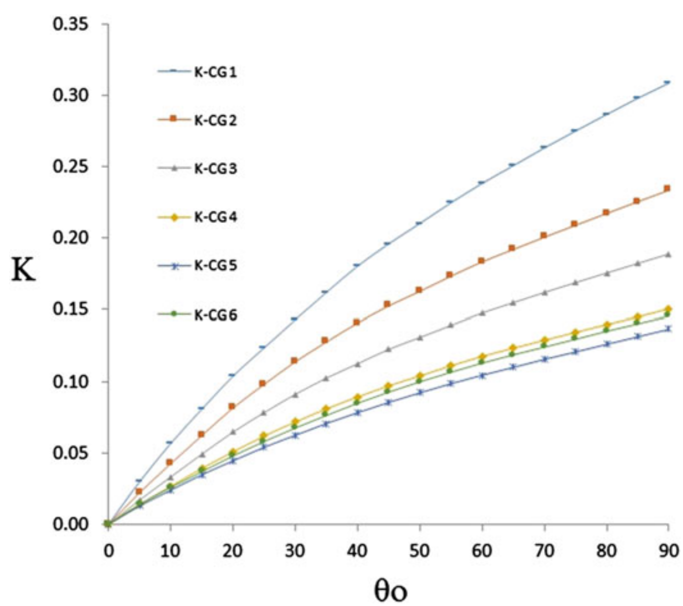

Fig. 4 Behavior of the average values in a gradual direction change of different relations between the ratio of the curve regarding the diameter of the conduit (*r/D*) according to the curve's deflection angle θ. Curves K-CG1, K-CG2, K-CG3, K-CG4, K-CG5, and K-CG6 correspond to the ratio r/D = 1, 1.5, 2, 4, 6, and 10, respectively

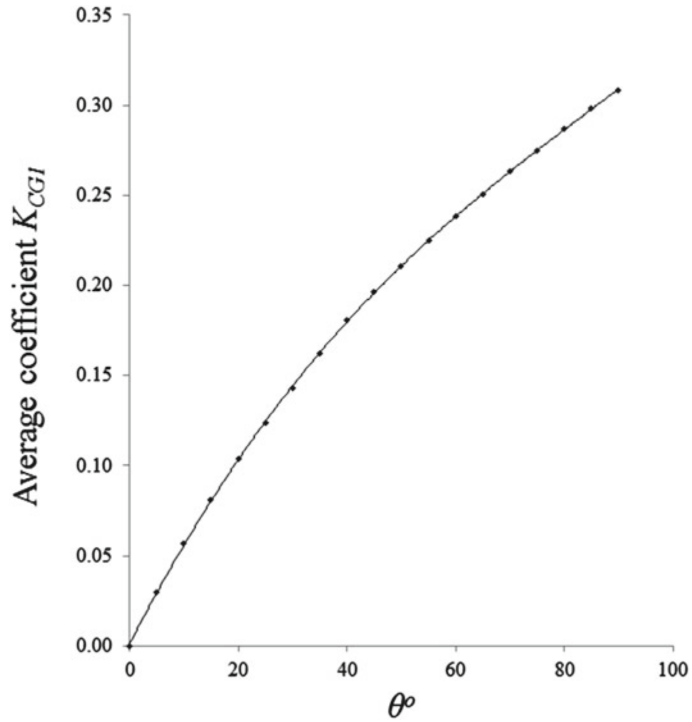

Fig. 5 Fitting curve to estimate the K_{CG1} coefficient as a function of the curve's deflection angle θ

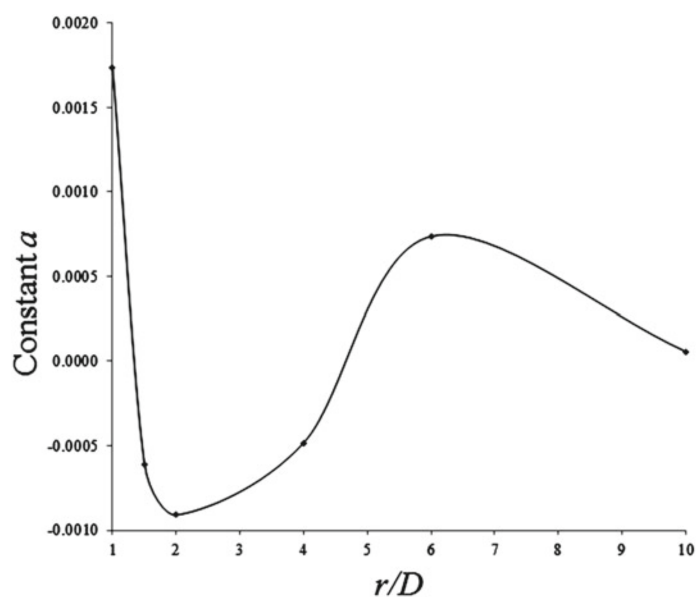

Fig. 6 Fitting curve of the constant *a* in Eq. 5 with respect between the curvature's ratio and the diameter of the conduit *r/D*

$$a = -0.0573379 + 0.00496834(r/D) - 0.00001716(r/D)^3 + \frac{0.07867083}{(r/D)^{0.5}} - \frac{0.066727}{e^{(r/D)}} \tag{6}$$

$$\mathrm{b} = 0.20495202 + 0.05446522(r/D) - 0.08723377(r/D)^{0.5}\ln(r/D) - \frac{0.45002930\ln(r/D)}{(r/D)} - \frac{0.25130468}{(r/D)^2} \tag{7}$$

$$c = -0.01383436 - 0.01385106(r/D) + 0.00051449(r/D)^2 + 0.04504019 ln(r/D) + \frac{0.08991395}{e^{(r/D)}} \tag{8}$$

Considering the equations for constants *a, b,* and *c,* Eq. (5) is presented as follows:

$$K_{CG} = \left\{ \left[-0.0573379 + 0.00496834(r/D) - 0.00001716(r/D)^3 + \frac{0.07867083}{(r/D)^{0.5}} - \frac{0.066727}{\mathrm{e}^{(r/D)}} \right] + \left[-0.01383436 - 0.01385106(r/D) + 0.00051449(r/D)^2 + 0.04504019\ln(r/D) + \frac{0.08991395}{\mathrm{e}^{(r/D)}}(\theta) \right] \right\} / \left\{ 1 + \left[0.20495202 + 0.05446522(r/D) - 0.08723377(r/D)^{0.5}\ln(r/D) - \frac{0.45002930\ln(r/D)}{(r/D)} + \frac{0.25130468}{(r/D)^2} \right](\theta) \right\} \tag{9}$$

where: K_{CG} is the losses coefficient in a gradual direction change; This equation should be used under the following conditions: $1 \leq r/D \leq 10$ and $5 \leq \theta \leq 90$. The aim of these conditions is to obtain results under a confidence of 99 %.

By running the Eqs. (4) and (9) the K_{AB} and K_{AG} coefficients are obtained, respectively, which are substituted in Eq. (1) order to obtain energy losses in a sudden change direction and gradual change direction, respectively, without using tables and graphs.

4 Conclusions

By reviewing the literature, we realized different methods are suggested in order to determine the losses coefficient (K) for each studied device. After analyzing such methods, it was determined that different values of *K* can be obtained with the use of one or other method; therefore, no standard values were registered. The implemented equations generate average *K* values of the analyzed methods with a determination of 0.99 and they not requires any handling of documents to determine its losses coefficient safely, exact and practical. Mathematical models implemented in this research can be applied jointly with Eq. (1) to obtain energy losses in the studied devices and save time in the design and testing of hydraulic behavior of forced conduits. Likewise, the equations can be implemented in advanced calculation algorithms that allow the analysis and modeling of losses caused by friction in different scenarios.

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