Rainfall Series Fractality in the Baja California State

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Abstract A fractal analysis of rainfall events registered in Baja California was carried out. Rainfall data from 92 climatological stations distributed along the studied region with at least 30 years of records were used. By studying rainfall series patterns, Hurst exponent values were obtained. The rescalated range method (R/S), box-counting method and the Multifractal Detrended Fluctuation Analysis (MF-DFA) were used, having as a result the Hurst exponent values for different time scales (entire record, 25, 10, and 5 years scales). Data showed that the daily rainfall series tended to present a persistent pattern. The analysis from the Hurst

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exponent on the previously mentioned time scales showed that, at a lesser time scale, their values increase; thus, the series tended to present a stronger persistent behavior.

1 Introduction

Rainfall is a random variable that evolves in time and space (Kandelhardt et al. 2003; Xu et al. 2015; Brunsell 2010; Gires et al. 2014; Millan et al. 2011; Pinel et al. 2014); therefore it can be analyzed spatially and temporally, allowing to comprehend the climatological behavior of a region based on its basic characteristics such as magnitude, duration and frequency.

Time series analysis studies a variable, in order to comprehend its evolution through time (Hoang et al. 2012); for which stochastic methods and fractal methods have been applied. Stochastic methods make predictions and obtain a relationship with other variables (Akbari and Friedel 2014; Caballero et al. 2002; Nunes et al. 2011); however, these methods present a disadvantage: a random variable, in this case rainfall, cannot be analyzed at different scales (Haui-Hsien et al. 2013; Svanidze 1980); therefore, fractal methods have been applied to analyze rainfall time series at multiple scales and dimensions (Lovejoy et al. 1987; Fluegeman and Snow 1989; Gallant et al. 1994; Lovejoy and Mandelbrot 1985; Lovejoy et al. 2012; Pinel et al. 2014; Schepers et al. 1992; Selvam 2010; Sivakumar 2000; Venugopal et al. 1999; Rangarajan and Sant 2004).

Mandelbrot (1967) introduced the concept of fractal in terms of self-similar statistics, stating that the shape of an object does not define its size; and established fractals as objects that possess a similar appearance when observed at different scales.

While determining the self similarity characteristics and the spatial-temporal fluctuations at multiple scales for rainfall time series, several methodologies have been applied; such as: rescalated range (R/S) (Hurst 1951, 1956; Mandelbrot 1972), box-counting method (Lovejoy et al. 1987; Breslin and Belward 1999), Multifractal Detrended Fluctuation Analysis (MF-DFA) (Movahed 2006), wavelet method (Velasquez et al. 2013), power spectrum method (Valdes-Cepeda et al. 2003), Higuchi method (Kalauzi et al. 2009), Detrended Fluctuation Analysis (DFA) (Yuval and Broday 2010), Fractal-Multifractal method (Huai-Hsien et al. 2013), Hurst-Kolmogorov method (Koutsoyiannis et al. 2011), Fractionally Integrated Flux (FIF) (Verrier et al. 2010), Fractal Brownian Surface (Tao and Barros 2010) and the fractal correlation dimension (D2) (Capecchi et al. 2012). Due to the application of the previous methods, it has been found that rainfall possesses fractal properties (Venugopal et al. 1999; Amaro et al. 2004; Beran 1994; Oñate 1997; Peters et al. 2002; Schertzer et al. 2010; Turcotte 1994; Lanza and Gallant 2006; Hubert and Carbonnel 1990); however the distribution of rainfall drops do not have these properties (Malinowki et al. 1993; Lovejoy and Schertzer 1987, 1990, 2006; Hentschel and Procaccia 1984; Lombardo et al. 2012).

Fractal behavior of rainfall time series has a relationship with altitude, such that at higher altitude a stronger antipersistent behavior occurs (Velasquez et al. 2013). A high frequency of zero values in the rainfall time series is an important factor to consider, as it can influence the fractal parameters obtained (Gires et al. 2012; Lopez-Lambraño 2012; Verrier et al. 2010).

The aim of this research is to analyze the fractal behavior of rainfall events in Baja California, Mexico, with the purpose of find a relation with climatological distribution, altitude above sea level, average temperature and average annual rainfall. So as to achieve this, 92 climatological stations with at least 30 years of records were analyzed; subsequently, using the Hurst exponents, a spatial and temporal analysis was carried out with the object of compare it with the previously mentioned variables. The expected results will allow to find a relation that enables to conduct climate change analysis in the studied zone.

2 Theory

2.1 Rescalated Range Method (R/S)

The original scheme proposed by Hurst (1951, 1956) proceeds with the following steps: the input data (daily rainfall) are taken as the difference between the rainfalls on two consecutive days:

$$I'_i = I_i - I_{i+1} \tag{1}$$

where I_i is the total rain amount on day i. The mean value of the difference over a period τ is:

$$\langle I' \rangle_\tau = \frac{1}{\tau} \sum_{i=1}^{\tau} I'_i \tag{2}$$

Now let $X(i, \tau)$ be the difference between I'_i and $\langle I' \rangle_\tau$ defined as

$$X(i, \tau) = \sum_{u=1}^{i} \left[I'_u - \langle I' \rangle_\tau \right] \tag{3}$$

Finally, the variable R and the standard variation S are given by

$$R(\tau) = \max_{1 \le i \le \tau} X(i, \tau) - \min_{1 \le i \le \tau} X(i, \tau) \tag{4}$$

$$S(\tau) = \left\{ \frac{1}{\tau} \sum_{i=1}^{\tau} \left[I'_i - \langle I' \rangle_\tau \right]^2 \right\}^{1/2} \tag{5}$$

We therefore obtain, for each value of the scale τ, an R/S number which follows a power law. If we take logarithms on both sides, we obtain the Hurst exponent value H as the value of the slope:

$$\log(R/S) \sim H \log(\tau) \tag{6}$$

2.2 Box-Counting Method

According to Peñate et al. (2013), the box counting method is based on dividing the space of observation (the time interval T) into n non-overlapping segments of characteristic size s, such that $s = T/n$ for $n = 2, 3, 4, \ldots,$ and computing the number $N(s)$ of intervals of length s occupied by events. If the distribution of events has a fractal structure, then the relationship $N(s) = Cs^D$ prevails. The fractal or box-counting dimension D_f is estimated from the slope D of the regression line of $\log(N(s))$ versus $\log(s)$. This parameter, $D_f = |D|$, describes the strength of the events and can measure the phenomenon's nature since it quantifies the scale-invariant clustering of the time series. Clustering increases when D approaches 0. Hence, the smaller fractal dimensions correspond to clusters formed by events that occur sparsely. If D is close to 1, the events are randomly spaced in time. To sum up, the box-counting method uses boxes to cover an object to find the fractal dimension. The signal is partitioned into boxes of various sizes and the amount of non-empty squares is counted. A log-log plot of the number of boxes versus the size of the boxes is done. The box dimension is defined by the exponent D_b in the relation:

$$N(s) \approx \frac{1}{s^{D_b}} \tag{7}$$

where $N(s)$ is the number of boxes with linear size, s needed to cover the set of points distributed on a bidimensional plane. A number of boxes, proportional to a $1/s$, are needed to cover the set of points on a line; proportional to $1/s^2$, to cover a set of points on a plane, etcetera.

In theory, for each box size, the grid should be configured in such a way that the minimum number of boxes is occupied. This can be achieved by rotating the grid for each box size in 90° and plotting the minimum value of $N(d)$.

The Hurst exponent value can be calculated from the following expression:

$$D_b = 2 - H \tag{8}$$

where H is the Hurst exponent value.

2.3 Multifractal Detrended Fluctuation Analysis

According to Yuval and Broday (2010) and Movahed et al. (2006), the modified multifractal DFA (MF-DFA) procedure consists of five steps. Suppose that x_k is a series of length N, and that this series is of compact support, i.e. $x_k = 0$ for an insignificant fraction of the values only.

Step 1—Determine the profile:

$$Y(i) = \sum_{k=1}^{i} [x_k - \langle x \rangle], \qquad i = 1, \ldots, N. \tag{9}$$

Subtraction of the mean $\langle x \rangle$ is not compulsory, since it would be eliminated by the later detrending in the third step.

Step 2—Divide the profile $Y(i)$ into $N_s = \mathrm{int}(N/s)$ non-overlapping segments of equal lengths s since the length N of the series is often not a multiple of the timescale s considered, a short part at the end of the profile may remain. In order to disregard this part of the series, the same procedure is repeated starting from the opposite end. Thereby, $2N_s$ segments are obtained altogether.

Step 3—Calculate the local trend for each of the $2N_s$ segments by a least squares fit of the series. Then determine the variance:

$$F^2(s, v) = \frac{1}{s} \sum_{i=1}^{s} \{Y[(v-1)s + i] - y_v(i)\}^2 \tag{10}$$

For each segment $v, v = 1, \ldots, N_s$, and:

$$F^2(s, v) = \frac{1}{s} \sum_{i=1}^{s} \{Y[N - (v - N_s)s + i] - y_v(i)\}^2 \tag{11}$$

For $v = N_s + 1, \ldots, 2N_S$. Here $y_V(i)$ is the fitting polynomial in segment v. Linear, quadratic, cubic or higher order polynomials can be used in the fitting procedure. Since the detrending of the time series is done by the subtraction of the polynomial fits from the profile, different order DFA differ in their capability of eliminating trends in the series.

Step 4—Average over all segments to obtain the qth-order fluctuation function, defined as

$$F_q(s) = \left\{ \frac{1}{2N_S} \sum_{v=1}^{2N_S} \left[F^2(s,v) \right]^{q/2} \right\}^{1/q} \tag{12}$$

Where, in general, the index variable q can take any real value except zero. For $q = 2$, the standard DFA procedure is retrieved. Generally we are interested in how the generalized q dependent fluctuation functions $F_q\ (s)$ depend on timescale s for different values of q. Hence, we must repeat steps 2, 3 and 4 for several timescales s. It is apparent that $F_q\ (s)$ will increase while increasing s. Of course $F_q\ (s)$ depends on the DFA order m. By construction, $F_q\ (s)$ is only defined for $s \geq m + 2$.

Step 5—Determine the scaling behavior of the fluctuation functions by analyzing log-log plots of $F_q\ (s)$ versus s for each value of q. If the series x_i are long range power law correlated, $F_q(s)$ increases, for large values of s, as a power law,

$$F_q(s) \sim s^{h(q)} \tag{13}$$

In general, the exponent $h(q)$ may depend on q. The exponent $h(2)$ is identical to the well-known Hurst exponent.

3 Methodology

The research takes place in Baja California, Mexico, comprised between the coordinates (32.713N, 114.723O), (32.527N, 117.141O), (28.095N, 115.364O), and (27.994N, 112.799O).

For the purpose of this research, data from 92 climatological stations distributed along the studied region with at least 30 years of rainfall records were analyzed.

In order to obtain the previously mentioned data, the use of two climatological systems was required: Rapid Climatological Information Extractor (ERIC III) and the National Climatological Database (Clicom); both of them give availability to daily national climate data.

In order to expand the panorama of this research, it has been chosen to utilize a series of time scales classified in the following scenarios:

(a) Hurst exponent value H calculated from the analysis of a time scale equally in duration to the total time series (n = Entire record duration)
(b) Hurst exponent value H calculated from the analysis of 25 years long time (n = 25 years).
(c) Hurst exponent value H calculated from the analysis of 10 years long time (n = 10 years).

(d) Hurst exponent value H calculated from the analysis of 5 years long time (n = 5 years).

After calculating these values, an average value, obtained from the three previously mentioned methods (Rescalated range (R/S) using Eq. (6), box-counting method using Eq. (7) and MF-DFA using Eq. (13), is calculated for each time scale. The results obtained from same time scales are averaged.

4 Results and Discussions

Given the importance to assess the fractal behavior of the daily rainfall series, and to analyze the variability of the Hurst exponent at different time scales, an analysis was carried out considering the previously mentioned time scales which correspond to: the Hurst exponent values H resulting from the analysis of the entire record for rainfall daily events, as well for the different time scales (25, 10, and 5 years long).

From Table 1, Hurst values behavior for daily rainfall events, at different time scales were reported. While analyzing the data, two mainly behaviors can be noticed for each of the time series in the analyzed variable: anti-persistent and persistent behavior. Both of these behaviors will be discussed below.

According to Table 1, the stations Presa Rodriguez, San Vicente, Tijuana, Valle de las Palmas, Presa Emilio Lopez Zamora, and Santo Tomas, which are located in the northeast region of Baja California, presented Hurst exponent values as follows: 0.45, 0.48, 0.49, 0.48. 0.47 y 0.47 respectively, i.e. they are anti-persistent in time. This means that rainfall events, which take place in this region, present a high probability of showing a positive increasing behavior, followed by a negative increasing behavior in its record values and vice versa.

According to Malamud and Turcotte (1999), an anti-persistent time series will have a stationary behavior in time; due to the increases and decreases that compensate themselves, statistical moments are independent from the time series. Rehman (2009) establishes that, in the case of rainfall, an anti-persistent behavior indicates a lesser dependency in accordance with previously stated values.

Nevertheless, when the rainfall series in the previously mentioned stations were analyzed in 25 years long scales, the Hurst values were reported as follows: 0.62, 0.65, 0.62, 0.63, 0.64 y 0.62, respectively. These values indicate a persistent behavior in time, meaning that, if the rainfall series registers a positive increase, it is more likely that a positive increase will follow. This implies that each rainfall event has a degree of occurrence over future events or in its long-term behavioral memory.

Continuing with the analysis of the previously mentioned stations, and considering the 10 and 5 years long time scales, increases in the Hurst values are noticeable once again and continuing with a persistent behavior throughout time. It can be found that the Hurst exponent increases while establishing shorter time scales.

Table 1 Averaged Hurst exponent results for climatological stations in Baja California for all time scales for daily rainfall events

#	Climatological station	Elevation above sea level (meters)	Average annual rainfall (mm/year)	Average annual temperature (°C)	Average Hurst exponent for the different time scales			
					Entire record	25 years	10 years	5 years
2001	Agua caliente	400	272.7	12.68	0.51	0.60	0.66	0.68
2002	Bahía de los ángeles	4	63.77	21.98	0.65	0.74	0.79	0.81
2003	Bataquez	23	69.33	20.02	0.61	0.74	0.78	0.82
2004	Ignacio Zaragoza	540	294.04	12.21	0.54	0.62	0.69	0.73
2005	Boquilla santa rosa de l	250	260.9	13.22	0.54	0.64	0.64	0.65
2006	Chapala	660	115.15	19.59	0.57	0.70	0.71	0.75
2008	Colonia guerrero	30	156	15.34	0.53	0.61	0.67	0.71
2009	Colonia Juárez	17	57.13	19.01	0.61	0.68	0.78	0.79
2011	Delta	12	46.65	19.76	0.65	0.72	0.77	0.80
2012	Ejido j. María Morelos	20	62.74	15.53	0.65	0.77	0.79	0.85
2014	El álamo	1115	265.7	13.66	0.52	0.60	0.66	0.72
2015	El arco	288	102.75	17.62	0.59	0.74	0.78	0.86
2016	El barril	50	81.31	24.12	0.61	0.76	0.78	0.85
2019	El compadre	1110	293.43	18.84	0.58	0.63	0.68	0.69
2020	El mayor	15	56.16	17.74	0.69	0.80	0.83	0.89
2021	El Pinal	1320	453.8	9.22	0.55	0.64	0.70	0.78
2022	El rosario	40	168.59	16.78	0.59	0.72	0.76	0.81
2023	El socorro	26	106.08	17.11	0.59	0.79	0.80	0.84
2024	El testerazo	380	240.57	11.72	0.65	0.72	0.76	0.76
2027	Isla cedros	3	62.72	19.74	0.71	0.81	0.84	0.87
2029	La providencia	40	257.24	13.93	0.57	0.72	0.73	0.78
2030	La puerta	480	326.51	13.56	0.51	0.64	0.69	0.70
2031	La rumorosa	1232	137.99	14.7	0.60	0.73	0.73	0.79
2032	Las escobas	30	132.73	15.03	0.51	0.70	0.75	0.80
2033	Mexicali (DGE)	3	72.98	18.8	0.57	0.75	0.76	0.81
2034	Mexicali (SMN)	3	95.18	20.24	0.60	0.72	0.77	0.88
2035	Ojos negros	680	222.32	12.73	0.50	0.61	0.66	0.69
2036	Olivares mexicanos	340	279.9	14.46	0.53	0.68	0.70	0.76
2037	Presa Morelos	40	61.4	17.83	0.61	0.66	0.74	0.82
2038	Presa Rodríguez	120	208.46	14.63	0.45	0.62	0.65	0.70
2039	Punta prieta	325	89.93	16.47	0.57	0.74	0.76	0.84
2040	Rancho alegre	120	120.32	15.83	0.59	0.75	0.75	0.82
2041	Nuevo rosarito	20	110.32	16.84	0.58	0.74	0.75	0.79
2043	San Agustín	552	111.09	17.13	0.57	0.75	0.76	0.81

(continued)

Table 1 (continued)

#	Climatological station	Elevation above sea level (meters)	Average annual rainfall (mm/year)	Average annual temperature (°C)	Average Hurst exponent for the different time scales			
					Entire record	25 years	10 years	5 years
2044	San Borja	445	104.9	18.03	0.57	0.73	0.76	0.85
2045	San Carlos	164	263.3	15.19	0.55	0.68	0.69	0.75
2046	San Felipe	10	64.9	22.5	0.66	0.84	0.86	0.94
2049	San Juan de Dios del norte	1280	362.9	12.09	0.52	0.58	0.61	0.65
2050	San Juan de Dios del sur	480	92.7	16.7	0.69	0.73	0.76	0.80
2051	San Luis baja california	440	124.9	17.82	0.62	0.81	0.83	0.89
2053	San miguel	60	187.8	19.44	0.62	0.70	0.71	0.77
2055	San Telmo	110	211.7	13.8	0.55	0.65	0.73	0.76
2056	San Vicente	1150	248.7	13.21	0.48	0.65	0.73	0.74
2057	Santa Catarina norte	317	137.6	14.53	0.51	0.60	0.62	0.66
2058	Santa Catarina sur	410	128.2	16.38	0.54	0.71	0.73	0.79
2059	Santa Clara	980	265.3	18.84	0.58	0.69	0.73	0.80
2060	Santa Cruz	400	105.4	15.57	0.52	0.63	0.66	0.71
2061	Santa Gertrudis	28	160.6	18.06	0.62	0.72	0.75	0.84
2063	Santa María del mar	250	211.5	14.95	0.54	0.71	0.73	0.79
2064	Santo domingo	180	259.14	14.06	0.58	0.72	0.74	0.79
2065	Santo tomas	20	215.6	16.43	0.47	0.62	0.65	0.67
2068	Tijuana	280	206.9	15.22	0.49	0.62	0.66	0.72
2069	Valle de las palmas	242	291.19	12.74	0.48	0.63	0.69	0.72
2070	Valle redondo	740	204.72	15.01	0.54	0.60	0.61	0.65
2071	Colonia valle de la trinidad	43	250.99	10.59	0.51	0.59	0.62	0.67
2072	Presa Emilio López Zamora	710	264.08	14.58	0.47	0.64	0.66	0.69
2079	El alamar	517	135.6	12.8	0.54	0.63	0.68	0.69
2084	El progreso	495	132.9	17.64	0.60	0.73	0.76	0.83
2085	San Regis	860	214.48	17.97	0.59	0.73	0.73	0.78
2086	Ejido Jacume	1000	263.6	13.24	0.58	0.67	0.72	0.74
2088	Ejido héroes de la Independencia	180	187.2	14.08	0.61	0.71	0.72	0.75
2089	Ejido Emilio López Zamora	170	269.9	14	0.68	0.75	0.75	0.78
2091	Ejido Ignacio López Ray	968	214.2	11.88	0.55	0.67	0.69	0.74

(continued)

Table 1 (continued)

#	Climatological station	Elevation above sea level (meters)	Average annual rainfall (mm/year)	Average annual temperature (°C)	Average Hurst exponent for the different time scales			
					Entire record	25 years	10 years	5 years
2092	Ejido san Matías	780	231.18	15.95	0.56	0.63	0.67	0.73
2093	Ejido valle de la trinidad	210	209.38	11.6	0.57	0.71	0.74	0.79
2096	La calentura	460	71.5	14.27	0.59	0.69	0.72	0.77
2099	Rancho los algodones	50	46.12	19.3	0.74	0.78	0.80	0.87
2101	El centinela	16	36.32	20.7	0.81	0.88	0.90	0.93
2102	La ventana	8	202.84	21.3	0.71	0.88	0.88	0.92
2104	El Ciprés	50	202.02	15.25	0.59	0.70	0.72	0.74
2106	Maneadero	600	120.7	15.28	0.57	0.69	0.73	0.73
2107	Percebu	4	40.7	19.12	0.78	0.87	0.90	0.92
2108	Punta banda	15	260.6	15.13	0.59	0.70	0.72	0.76
2109	Santa Rosalita	8	136.02	16.64	0.69	0.78	0.82	0.87
2110	Guayaquil	530	123.5	17	0.65	0.72	0.77	0.80
2111	Ejido nueva baja california	17	134.9	14.13	0.62	0.72	0.74	0.77
2114	Ejido Carmen Serdán	560	231.8	11.57	0.62	0.72	0.74	0.77
2118	Valle san Rafael	721	218.4	12.45	0.54	0.66	0.69	0.71
2120	Ejido México	75	176.9	13.61	0.64	0.71	0.75	0.78
2121	El hongo	960	286.35	12.55	0.52	0.61	0.63	0.67
2124	El carrizo ii	300	231.88	13.59	0.53	0.63	0.65	0.68
2137	Colonia mariana	9	57.61	19.4	0.65	0.75	0.78	0.82
2139	Colonia Rodríguez	17	36.8	18	0.73	0.82	0.88	0.90
2140	Colonia Zaragoza	8	65.16	12.43	0.86	0.82	0.88	0.94
2141	Compuerta Benassini	20	55.11	17.9	0.63	0.75	0.80	0.84
2144	Ensenada blanca	10	89.07	19.7	0.69	0.75	0.82	0.84
2145	Rancho Williams	29	63.05	20.57	0.61	0.73	0.77	0.83
2146	Colonia san pedro mártir	416	106.3	16.19	0.60	0.74	0.75	0.81
2151	Agua de chale	5	53.12	22.59	0.68	0.82	0.83	0.88
2152	Ejido j. María del pino	1380	273.6	8.7	0.62	0.72	0.74	0.75
2153	Ejido Uruapan	195	288.55	13.9	0.61	0.70	0.72	0.75
2154	Colonia zacatecas	12	52.12	18.46	0.71	0.80	0.81	0.87

Anti-persistent series kept a positive increase tendency with minor negative increases; thus, while considering shorter periods of times (25, 10, 5 years), it is reasonable that series starts to behave in a more persistent manner. This is due to the fact that, the shorter the considered time scale is, the greater the possibility to analyze a time period with a predominant behavior that, in this case, is a persistent behavior; this could explain the increase in the Hurst values in the analyzed rainfall series.

Continuing the analysis on the other 86 remaining stations and considering the established time scales, increases in the Hurst values are noticeable, keeping a persistent behavior tendency since the values are greater than 0.5.

This means that rainfall series is not stationary and that moments are time scale dependent. The presence of persistency is an indicator that, not only what takes place in the present influence the near future, it will also influence long term future.

5 Conclusions

Even though three methods (Multifractal Detrended Fluctuation Analysis, box-counting method, and rescalated range (R/S)) were utilized to calculate the Hurst exponent for each analyzed scale time, it is difficult to determine which of these methods is the most efficient; however, the rescalated range method has been reported as the most used method.

Registered daily rainfall series throughout Baja California can be characterized by using the Hurst exponent, having as a result a tendency to present a persistent behavior. Thus, when a positive increase is registered, it is more likely that the following increase will be positive as well.

The existence of dependency between the rainfall and temperature climate variables in relation with altitude can be noticed. Regions with an altitude near to sea level tend to register the highest temperature values, as well as the lowest rainfall annual average. It can be established that a proportional inverse relation between altitude and temperature exists, as well as a proportional direct relation between altitude and rainfall.

Taking into account the Hurst exponent used in this research, a proportional inverse relation with altitude and rainfall can be noticed; in regions that are close to sea level, high values can be reported; on the other hand, regions where the average annual rainfall is high, the analysis showed low values for the Hurst exponent. It can be confirmed that the Hurst exponent depends in the climatological conditions and physiographic characteristics in a specific region.

Taking into account shorter time scales (25, 10, 5 years long in this research), it can be found that the series' persistency is stronger. The greater the number of considered scale times to analyze rainfall series, the greater the possibility to understand its behavior and tendencies.

Finally, it can be confirmed that fractal theory provides information that allows to analyze the occurrence of a climatological variable, such as rainfall (in the case for this research). This provides a useful tool to study and mitigate climate change in a given region.

It is recommended to use multifractal theory in future researches, having as a main purpose the chance to study the scale invariability from a mathematical perspective and to describe the climatic variables behavior with potential laws which are characterized by its exponents.

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